In Exercises 1–11, using induction, verify that each equation is true for every positive integer n.

1. $1 + 3 + 5 + \cdots + (2n - 1) = n^2$

2. $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n + 1) = \dfrac{n(n + 1)(n + 2)}{3}$

3. $1(1!) + 2(2!) + \cdots + n(n!) = (n + 1)! - 1$

4. $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n + 1)(2n + 1)}{6}$

5. $1^2 - 2^2 + 3^2 - \cdots + (-1)^{n+1}n^2 = \dfrac{(-1)^{n+1}n(n + 1)}{2}$

6. $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\dfrac{n(n + 1)}{2}\right]^2$

7. $\dfrac{1}{1 \cdot 3} + \dfrac{1}{3 \cdot 5} + \dfrac{1}{5 \cdot 7} + \cdots + \dfrac{1}{(2n - 1)(2n + 1)}$

$$= \frac{n}{2n + 1}$$

8. $\dfrac{1}{2\cdot 4}+\dfrac{1\cdot 3}{2\cdot 4\cdot 6}+\dfrac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}+\cdots+\dfrac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 4\cdot 6\cdots(2n+2)}$

$$=\frac{1}{2}-\frac{1\cdot 3\cdot 5\cdots(2n+1)}{2\cdot 4\cdot 6\cdots(2n+2)}$$

9. $\dfrac{1}{2^2-1}+\dfrac{1}{3^2-1}+\cdots+\dfrac{1}{(n+1)^2-1}$

$$=\frac{3}{4}-\frac{1}{2(n+1)}-\frac{1}{2(n+2)}$$

⋆10. $\cos x+\cos 2x+\cdots+\cos nx=\dfrac{\cos[(x/2)(n+1)]\sin(nx/2)}{\sin(x/2)}$

provided that $\sin(x/2)\neq 0$.

⋆11. $1\sin x+2\sin 2x+\cdots+n\sin nx$

$$=\frac{\sin[(n+1)x]}{4\sin^2(x/2)}-\frac{(n+1)\cos[(2n+1)x/2]}{2\sin(x/2)}$$

provided that $\sin(x/2)\neq 0$.