Mathematical Induction:

In order to prove that S(n) is true for all n, we have to prove the following two steps.

Step 1: prove that S(1) is true .

Step 2: prove that "If S(k) is true, then S(k+1) is true where $k \geq 1$." (This is a promise.)

Under the condition S(k) being true, if we can prove S(k+1) being true, then the promise is done. (Think that if p then q. If p is true then q must be true, otherwise, the statement if p then q will be false.)

So, you have to identify your S(n), S(1), S(k), and S(k+1) in order to solve. You also need to use S(k) information to prove S(k+1) being true.

Example:

$$S(n) \equiv "1(1!) + 2(2!) + \cdots + n(n!) = (n+1)! - 1, \forall n"$$

Step 1: prove that S(1) is true. (Target)

Let $n = 1$, then $S(1) \equiv "1(1!) = (1+1)! - 1"$ (need to be proved)

Since $1(1!) = 1 \cdot 1 = 1$ and $(1+1)! - 1 = 2! - 1 = 2 - 1 = 1$,

we have $S(1)$ being true.

Step 2: prove that "If S(k) is true, then S(k+1) is true where $k \geq 1$." (Target)

Identify $S(k)$ and $S(k+1)$ first.

Let $k \geq 1$, $S(k) \equiv "1(1!) + 2(2!) + \cdots + k(k!) = (k+1)! - 1"$ and

$S(k+1) \equiv "1(1!) + 2(2!) + \ldots + k(k!) + (k+1)[(k+1)!] = [(k+1)+1]! - 1"$

Assume $S(k)$ being true, we want to prove $S(k+1)$ being true.

Use the information of $S(k)$ to prove $S(k+1)$.

Plot in the information of $S(k)$ into the left side of equal sign of $S(k+1)$.

So, replace $1(1!) + 2(2!) + \cdots + k(k!)$ by $(k+1)! - 1$ on the left side of equal sign of $S(k+1)$. After taking this replacement, you have the left side as

$(k+1)! - 1 + (k+1)[(k+1)!]$

$= 1 \cdot [(k+1)!] + (k+1)[(k+1)!] - 1$ (Exchange the order of terms)

$= (1 + k + 1)[(k+1)!] - 1$ (Move out the common factor)

$= (k+2)[(k+1)!] - 1 = (k+2)! - 1$

It is same as the right side of $S(k+1)$ which is

$[(k+1)+1]! - 1 = (k+2)! - 1$

Therefore, $S(k+1)$ is true. We conclude that $S(k) \rightarrow S(k+1)$ is true.

We can conclude that

$S(n) \equiv "1(1!) + 2(2!) + \cdots + n(n!) = (n+1)! - 1, \forall n"$ is true.