Show your work.

1. Concern an unprepared student who takes a 10-question true-false quiz and guesses at the answer to every question. What is the probability that the student answers every question correctly?

$$\left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$$

2. Concern an unprepared student who takes a 10-question true-false quiz and guesses at the answer to every question. What is the probability that the student answers exactly one question correctly?

$$10 \times \left(\frac{1}{2}\right)^{10} = \frac{5}{512}$$

3. If a family has four children, what is the probability of all girls? Assume that it is equally probable for a boy or a girl to be born.

$$\left(\frac{1}{2}\right)^{4} = \frac{1}{16}$$

4. If a family has four children, what is the probability of all girls given that there is at least one girl? Assume that it is equally probable for a boy or a girl to be born.

$$P(all\ girls\ |at\ least\ one\ girl) = \frac{P(all\ girls \cap at\ least\ one\ girl)}{P(at\ least\ one\ girl)}$$

$$= \frac{P(all\ girls)}{P(at\ least\ one\ girl)} = \frac{\frac{1}{16}}{1-\frac{1}{16}} = \frac{1}{15}$$

5. Six microprocessors are randomly selected from a lot of 100 microprocessors among which 10 are defective. Find the probability of obtaining no defective microprocessors.

|sample|=C_6^{100}, |Event|=C_6^{90}, P=$\frac{C_6^{90}}{C_6^{100}}$=$\frac{90 \cdot 89 \cdot 88 \cdot 87 \cdot 86 \cdot 85}{100 \cdot 99 \cdot 98 \cdot 97 \cdot 96 \cdot 95}$

6. Six microprocessors are randomly selected from a lot of 100 microprocessors among which 10 are defective. Find the probability of obtaining at least one defective microprocessor.

$$1 - the\ answer\ of\ problem5 = 1 - \frac{C_6^{90}}{C_6^{100}}$$

7. Six microprocessors are randomly selected from a lot of 100 microprocessors among which 10 are defective. Find the probability of obtaining at least three defective microprocessors.

|not event|=$C_6^{90} + C_5^{90}C_1^{10} + C_4^{90}C_2^{10}$, P=$1 - \frac{C_6^{90}+C_5^{90}C_1^{10}+C_4^{90}C_2^{10}}{C_6^{100}}$

8. Two dice are rolled. What is the probability of getting a sum of 6 or a sum of 8?

Events={(1,5),(5,1),(2,4),(4,2),(3,3),(2,6),(6,2),(3,5),(5,3),(4,4)}

$$P = \frac{10}{6 \times 6} = \frac{5}{18}$$

9. Consider the following events in the toss of a single die.
   E: Observe an odd number.
   F: Observe an even number.
   G: Observe a 1 or 2.
   a. Are E and F independent events?

$$P(E) = \frac{1}{2} \qquad P(F) = \frac{1}{2} \qquad P(E \cap F) = 0 \neq P(E) \cdot P(F)$$

$$So, it\ is\ not\ independent.$$

   b. Are E and G independent events?

$$P(E) = \frac{1}{2} \qquad P(G) = \frac{1}{3} \qquad P(E \cap G) = \frac{1}{6} = P(E) \cdot P(G)$$

   So, it is independent.

   c. Are E and F mutually exclusive events? Yes.

   d. Are E and G mutually exclusive events?

No, $E \cap G \neq \emptyset$.