I. Determine whether the relation of $\{(\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{R} \textbf{ if } \boldsymbol{x} \geq \boldsymbol{y}\}$ is ***reflexive***, ***symmetric***, ***antisymmetric***, ***transitive***, and/or ***a partial order***.

II. Let X be the set of all four-bit strings (e.g., 0011, 0101, and 1000). Define a relation R on X as $s_1 R\, s_2$ if some substring of s_1 of length 2 is equal to some substring of s_2 of length 2. Example: $0111\ R\ 1010$ (because both 0111 and 1010 contain 01). $1110 \ \backsim R\ 0001$ (because 1110 and 0001 do not share a common substring of length 2). Is this relation ***reflexive***, ***symmetric***, ***antisymmetric***, ***transitive***, and/or ***a partial order***?

III. Let R_1 and R_2 be the relations on {1, 2, 3, 4} given by

$R_1 = \{(1,1), (1,2), (3,4), (4,2)\}$

$R_2 = \{(1,1), (2,1), (3,1), (4,4), (2,2)\}$

List the elements of $\boldsymbol{R_1 \circ R_2}$ and $\boldsymbol{R_2 \circ R_1}$.

IV. Determine whether the given relation is an **equivalence relation** on $\{1, 2, 3, 4, 5\}$. If the relation is an **equivalence relation**, list the **equivalence classes**.

(a) $\{(\mathbf{1},\mathbf{1}), (\mathbf{2},\mathbf{2}), (\mathbf{3},\mathbf{3}), (\mathbf{4},\mathbf{4}), (\mathbf{5},\mathbf{5}), (\mathbf{1},\mathbf{3}), (\mathbf{3},\mathbf{1}), (\mathbf{3},\mathbf{4}), (\mathbf{4},\mathbf{3})\}$

(b) $\{(\boldsymbol{x}, \boldsymbol{y}) |\ \mathbf{4 \text{ divides }} \boldsymbol{x} - \boldsymbol{y}\}$

V. $R_1 = \{(x, y) |\ x + y \ \leq 6\}$; R_1 is from X to Y;

$R_2 = \{(y, z) | y = z + 1\}$; R_2 is from Y to Z;

$X = Y = Z = \{1, 2, 3, 4, 5\}$

(a) Write each of the relations R_1 and R_2 in the **matrix forms**. (A_1 and A_2)

(b) Find the **matrix product** $\boldsymbol{A_1 A_2}$.

(c) Use the product matric to write the **relation of** $\boldsymbol{R_2 \circ R_1}$.